

Simulation Studies on an Asymmetrically Heated Vertical Channel Involved in Multi-Mode Heat Transfer

S. Sudhakar Babu¹, C. Gururaja Rao² and A.V. Narasimha Rao³

^{1,2,3}Dept. of Mechanical Engg. N.I.T., Warangal Telangana, India
E-mail: ¹sudha.227@gmail.com, ²cgr_gcr@yahoo.co.in, ³avnrao5@yahoo.co.in

Abstract—Selected results of simulation studies performed on an asymmetrically heated vertical channel taking part in multi-mode heat transfer are presented in the paper. A total of ten identical discrete heat sources have been flush-mounted in the channel with five per each of its walls. The equations for fluid flow and heat transfer governing the problem are converted into vorticity-stream function formulation before being non-dimensionalised. The resulting equations are discretized into algebraic form employing finite volume method. The cooling medium considered is air that is treated to be transparent to radiation. An explicit computer code is prepared to solve the problem considering pertinent boundary conditions on stream function, vorticity and temperature. Parametric studies exploring the effects of parameters like aspect ratio, emissivity, modified Richardson number and thermal conductivity on certain fluid flow and heat transfer results are performed. Interactive roles exhibited by buoyancy and radiation are extracted.

Keywords: Simulation studies, Vertical channel, Asymmetric heating, Multi-mode heat transfer

1. INTRODUCTION

Literature on multi-mode heat transfer contains several analytical, numerical and experimental studies involving interplay between conduction, convection and radiation from different kinds of geometries with various complexities. The first ever reported work of this kind is credited to Elenbaas [1], who, in his benchmark paper, reported the experimental results of free convection in a vertical symmetrically heated isothermal channel with air as the cooling agent. Following this, several other researchers came out with their results on problems of this kind. Quintiere and Mueller [2] documented their approximate analytical solutions for constant-property laminar mixed convection between finite vertical parallel plates. Watson et al. [3] numerically modeled laminar steady state buoyancy aided mixed convection between a series of vertical parallel plates taking wall conduction into reckoning. Barletta and Zanchini [4] studied, analytically, the fully developed laminar mixed convection of a Newtonian fluid with temperature dependent viscosity in an inclined plane channel with prescribed wall temperatures. Subsequently, the

geometry of vertical channel with symmetric and asymmetric uniform heat generation in the two walls has also been tackled by Gururaja Rao et al. [5]. Li et al. [6] documented numerical solutions regarding the effect of surface radiation on laminar air flow taking place in a vertical and asymmetrically heated channel.

A review of the literature concerning multi-mode heat transfer from different geometries typically encountered in electronic cooling applications suggests that detailed simulation studies on asymmetrically heated vertical channel involved in multi-mode heat transfer are not adequately explored. In view of this, the present paper takes up a detailed numerical investigation into the problem of conjugate mixed convection with surface radiation from an asymmetrically heated vertical channel.

2. PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

Few of the prominent simulation results of a numerical study on conjugate mixed convection with surface radiation from a vertical channel with multiple asymmetrically positioned discrete heat sources are elucidated in this paper. Fig. 1 shows a vertical, parallel-plate channel equipped with a total of ten discrete heat sources in its walls. The channel is of height L having its walls sufficiently thin (thickness $t \ll L$), while the spacing or width (W) varies in accordance with the aspect ratio (AR) that is defined as L/W . The above means that a smaller AR makes the walls of the channel wider apart with a larger AR rendering the channel narrower. There are five heat sources in each wall flush-mounted asymmetrically as shown. It may be noted that the heat sources are of identical size ($L_n \times t$) and have a volumetric heat generation q_v . The thermal conductivity of the wall material is taken to be k_s , while the surface emissivity is assumed to be ϵ . Air, a radiatively transparent medium, is the cooling agent that enters the channel from its bottom end at characteristic velocity u_∞ and temperature T_∞ . It is assumed to be of constant thermophysical properties subject to the Boussinesq approximation. As can be

seen from the figure, the top, left and bottom surfaces of the left wall and the top, right and bottom surfaces of the right wall are adiabatic implying that the heat generated in the ten heat sources possessed by the walls gets exchanged between the interior surfaces of the two walls via the intervening medium (air).

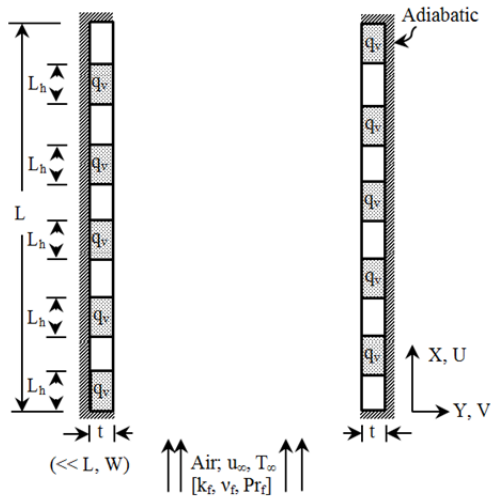


Fig. 1: Schematic of the asymmetrically heated vertical channel considered for study along with system of coordinates

The fundamental equations governing fluid flow and heat transfer are the continuity equation, the Navier-Stokes equations and the equation of energy, as shown below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \left(g \frac{\rho_\infty}{\rho} - g \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

These equations are first transformed from primitive variables to vorticity-stream function (ω - ψ) form and are later normalized. The governing equations for temperature distribution along the two walls of the channel are deduced through an appropriate energy balance between the heat generated, conducted, convected and radiated. Considering a typical element concerning the heat source portions (excluding their ends) pertaining to the left wall of the channel, the energy balance yields:

$$q_{\text{gen}} + q_{\text{cond},x,\text{in}} = q_{\text{cond},x,\text{out}} + q_{\text{conv}} + q_{\text{rad}} \quad (5)$$

Substituting the relevant expressions for various terms in the above equation and then simplifying the same, one gets:

$$k_s t \frac{\partial^2 T}{\partial x^2} + k_f \left(\frac{\partial T}{\partial y} \right)_{y=0} + q_v t - \frac{\epsilon}{1-\epsilon} [\sigma T_i^4 - J_i] = 0 \quad (6)$$

Similar equations as above are obtained for the rest of the portions of the two channel walls and are subsequently normalized.

3. METHOD OF SOLUTION AND PARAMETERS EMPLOYED

The normalized governing equations are converted into algebraic form using finite volume method. The resulting equations are solved using Gauss-Seidel iterative method. The computational domain is discretized considering cosine function in the transverse (Y) direction that yields relatively finer grids nearer the walls with the grids getting coarser towards the centre. The above helps in catering to steeper velocity and temperature gradients adjacent to the walls of the channel. Along the axial (X) direction, finer uniform grids are chosen for the walls of the channel. The open boundaries of the computational domain are assumed to be black ($\epsilon = 1$). A computer code in C++ is specifically prepared for solving the current problem incorporating all the above features. The ranges of all the independent parameters (k_s , ϵ , Ri_w^* and AR) are appropriately fixed before proceeding on to the simulation studies.

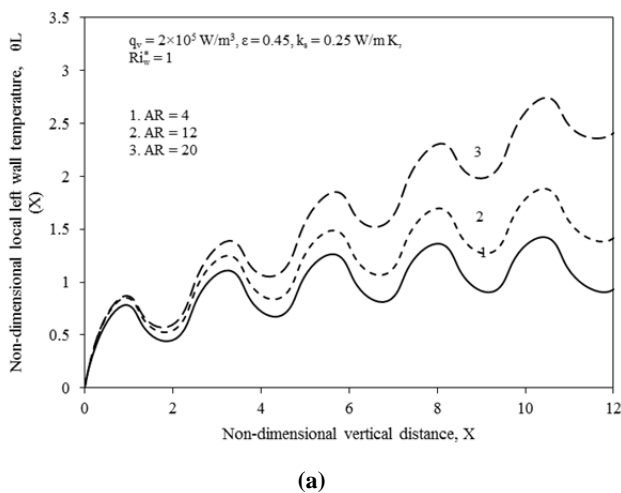
As mentioned already, all the calculations in the present study are performed making use of air ($Pr_f = 0.71$) as the cooling agent assuming it to be a radiatively transparent and Boussinesq medium. The height of the channel (L) is chosen to be 20 cm, while its thickness (t) is taken to be 1.5 mm. The height (L_h) of each discrete heat source is taken equal to $L/10$ (2 cm). With the height (L) of the channel fixed as above, its width or spacing (W) is a variable depending on the aspect ratio ($AR = L/W$), which is an important independent parameter in the present problem. The range of AR is taken to be from 4 to 20. For surface emissivity (ϵ) of the channel, a range of 0.05-0.85 is chosen with the two limiting values, respectively, indicating good reflector (polished aluminum) and good emitter (black paint). With regard to thermal conductivity (k_s) of the material of the channel, a range of 0.25-1 W/m K is considered as appropriate since the electronic boards are generally made of materials of thermal conductivity of the order of unity (epoxy glass, for example, has a thermal conductivity of 0.26 W/m K). For the modified Richardson number (Ri_w^*), the range is varied between 0.01 and 250, with the lower limit ($Ri_w^* = 0.01$) implying the asymptotic forced convection limit, while the upper limit ($Ri_w^* = 250$) signifies the asymptotic free convection limit.

4. PARAMETRIC STUDIES AND INFERENCES DRAWN

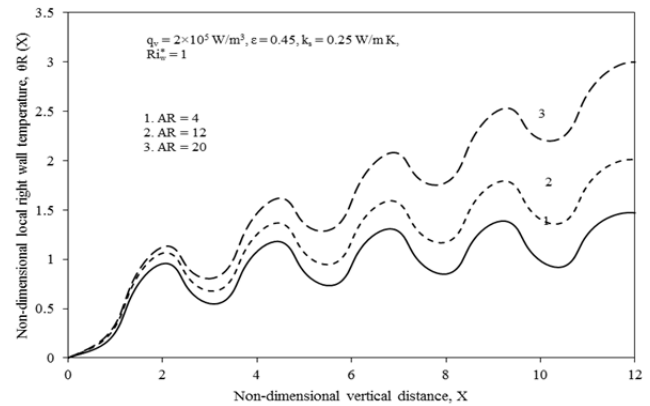
Variation of non-dimensional local wall temperature with aspect ratio of the channel

In order to establish the effect of aspect ratio (AR) on the local temperature distribution in both the channel walls, a study is

performed using a fixed input of $k_s = 0.25 \text{ W/m K}$, $q_v = 2 \times 10^5 \text{ W/m}^3$, $Ri_w^* = 1$ and $\varepsilon = 0.45$. It can be noticed from Fig. 2(a) that, for a given AR, the local temperature of the left wall $[\theta_L(X)]$ increases very sharply from the channel entry, reaches a local maximum somewhere near the end of the bottommost heat source. It subsequently decreases in the immediately succeeding non-heat source portion. After reaching a local minimum, it shoots up again as one moves through the second heat source exhibiting a similar behavior as noticed in the first heat source portion. The trend continues in the rest of the wall and one can see five local maxima and five local minima in the temperature profile. This nature of variation, understandably, is on account of the discreteness in heat generation in the wall with heat source and non-heat source portions present alternately. The largest of the five local peaks, which, incidentally, is the maximum left wall temperature of the channel, occurs in the fifth heat source from the bottom end of the left wall. The Fig. further shows that the local temperature increases with increasing AR from 4 to 20 (i.e., the channel progressively getting narrower). The above is attributed to an enhanced radiation exchange owing to reduced spacing between the channel walls. In the present case, the maximum non-dimensional left wall temperature of the channel is seen to rise by 92.49% as aspect ratio increases from 4 to 20. Fig. 2 (b) concerning the right wall hints at a similar nature of variation of local temperature $[\theta_R(X)]$ along it as that observed along the left wall with the waviness in the profile getting shifted in tune with the changed positions of the heat sources in it.



Here too, an increasing AR, expectedly, tends to increase the local temperature. In the specific case considered here, there is a 103.89% increase in $\theta_R(X)$ at the channel exit with AR increasing from 4 to 20. The above findings prohibit the designer from unduly packing the channel walls closer, for a given height, since it leads to an unwanted increase in the peak channel temperature.



(b)
Fig. 2: Local non-dimensional temperature profiles for (a) left wall (b) right wall for different aspect ratios of the channel

5. VARIATION OF MAXIMUM CHANNEL TEMPERATURE WITH SURFACE EMISSIVITY IN DIFFERENT REGIMES OF MIXED CONVECTION

Fig. 3 demonstrates the effect of surface emissivity (ε) in controlling the maximum non-dimensional temperature of the channel (θ_{max}) in various regimes of mixed convection. Five different values of ε and Ri_w^* are chosen as shown in the Fig. that also gives the remaining input parameters held fixed in the study. Here, $Ri_w^* = 250$ represents the asymptotic free convection limit, while $Ri_w^* = 0.01$ indicates the asymptotic forced convection limit. It is observed that θ_{max} decreases with increasing ε in all the regimes of mixed convection, with the degree of decrease getting more pronounced in the free convection dominant regime when compared to the forced convection dominant regime. This is due to an enhanced convection activity in the forced convection dominant regime that overrides dissipation by radiation.

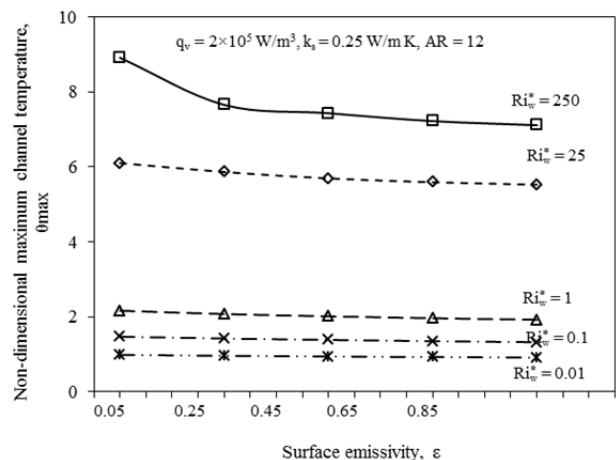


Fig. 3: Variation of non-dimensional maximum temperature of the channel with surface emissivity in different regimes of mixed convection

For example, in the study performed here, θ_{max} comes down by 20.19% and 7.38%, respectively, for $Ri_w^* = 250$ and 0.01, with ϵ increasing from 0.05 to 0.85. The Fig. further reveals that, for a given surface coating (ϵ), the peak channel temperature diminishes with decreasing Ri_w^* . This is due to the transit of flow regime from free convection dominance to forced convection dominance leading to enhanced convective heat dissipation that brings down θ_{max} . In particular, one can notice a far larger decrement in θ_{max} between $Ri_w^* = 250$ and $Ri_w^* = 1$ than that between $Ri_w^* = 1$ and $Ri_w^* = 0.01$. Quantifying the above observation, for a moderately emitting surface ($\epsilon = 0.45$), θ_{max} is seen to coming down by 72.91% owing to a change in Ri_w^* from 250 to 1, while a similar attempt of decreasing Ri_w^* further down to 0.01 from 1 is bringing only a 53.16% drop in θ_{max} . These findings provide the designer the needed choice in controlling the peak temperature attained by the channel either by appropriately selecting the surface coating in a given operating regime or by regulating the induced velocity of flow for a given surface coating.

6. EXCLUSIVE EFFECT OF RADIATION ON PEAK CHANNEL TEMPERATURE IN THE ENTIRE CONVECTION REGIME

A study bringing out the exclusive role of ϵ in controlling the maximum non-dimensional channel temperature over the entire mixed convection regime has been performed and the results are depicted in Fig. 4. Two values of ϵ are considered, viz., $\epsilon = 0$ (radiation absent) and $\epsilon = 0.99$ (maximum possible radiation), while the rest of the input includes $AR = 12$, $k_s = 0.25$ W/m K and $q_v = 2 \times 10^5$ W/m³. As many as seven values of Ri_w^* covering the entire regime of mixed convection have been chosen. The Fig. clearly underlines the fact that non-consideration of radiation in calculations leads to an undue overestimation of the peak channel temperature (θ_{max}) in all regimes of mixed convection, in general, and in the free convection dominant regime, in particular.

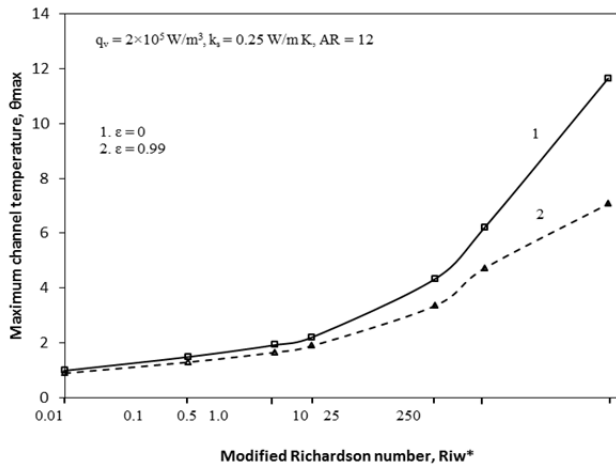


Fig. 4: Exclusive effect of radiation on peak channel temperature in different regimes of mixed convection

This occurs to a lesser degree for $Ri_w^* = 0.01$ and is found to increase gradually with increasing Ri_w^* , with the most perceivable effect noticed at $Ri_w^* = 250$. In the present study, θ_{max} comes down by 9.44% in the asymptotic forced convection limit ($Ri_w^* = 0.01$), upon changing the channel surface from a perfect reflector ($\epsilon = 0$) to a perfect emitter ($\epsilon = 0.99$). In the asymptotic free convection limit ($Ri_w^* = 250$), the above exercise of changing the surface of the channel brings a very significant drop in θ_{max} by as much as 39.29%. These observations underline the significant role exhibited by radiation in cutting down the maximum non-dimensional channel temperature in all the regimes of mixed convection.

7. RELATIVE CONTRIBUTIONS OF MIXED CONVECTION AND RADIATION WITH SURFACE EMISSIVITY IN DIFFERENT CONVECTION REGIMES

The heat generated in the channel due to the presence of the ten discrete heat sources in its walls is dissipated by mixed convection and surface radiation. It is appropriate to dwell into the individual contributions from mixed convection and radiation for various values of emissivity (ϵ) and in different regimes of mixed convection. Fig. 5, plotted for $q_v = 2 \times 10^5$ W/m³, $k_s = 0.25$ W/m K and $AR = 12$, reveals the findings of such a study. Five typical values of surface emissivity encompassing the range ($0.05 \leq \epsilon \leq 0.85$) and four different values of Ri_w^* , catering to the entire range of mixed convection from the free convection dominant regime ($Ri_w^* = 250$) to the forced convection dominant regime ($Ri_w^* = 0.01$) are used.

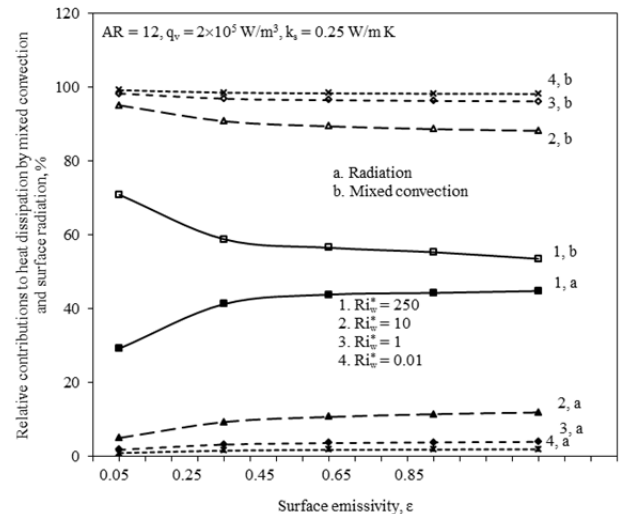


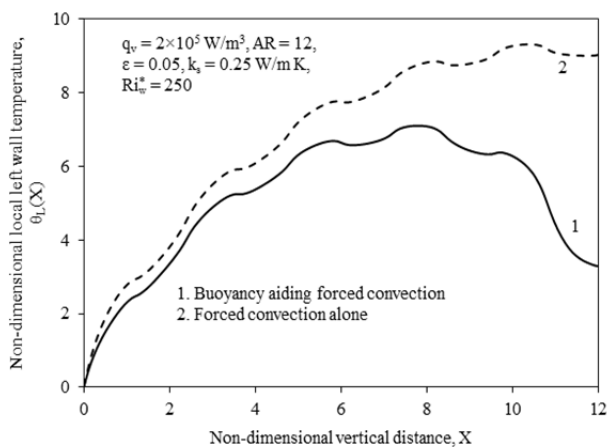
Fig. 5: Relative contributions of mixed convection and surface radiation in channel heat dissipation for different surface emissivities and in different regimes of mixed convection

An increasing ϵ is observed to bring down the contribution from convection with a proportionate increase in that from radiation. In the present example, for $Ri_w^* = 0.01$, i.e., the asymptotic forced convection limit, the contribution from radiation rises from 0.81% to 1.83% as ϵ increases from 0.05

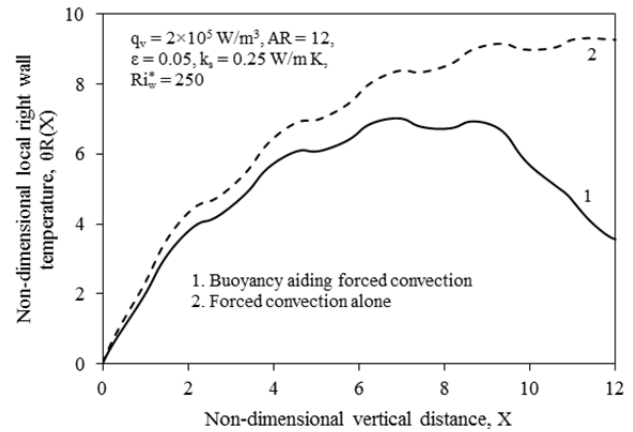
to 0.85. A similar exercise in the asymptotic free convection limit ($Ri_w^* = 250$) increases the radiative dissipation from 29.16% to 44.75%. There is a corresponding drop in the contribution from mixed convection in both the cases. It can also be noted that, for a given ε , as one moves from the forced convection dominant regime to the free convection dominant regime, the contribution of convection, expectedly, decreases with a mirror-image increase in the radiation contribution. For a good emitter (black paint with $\varepsilon = 0.85$), the convective heat dissipation is decreasing from 98.17% to 53.47% as Ri_w^* is changing from 0.01 to 250. It goes without saying that the role of radiation readjusts itself appropriate to the changes in the role of mixed convection owing to the above change in the flow regime (or Ri_w^*).

8. EXTRACTION OF EXCLUSIVE EFFECT OF BUOYANCY ON LOCAL TEMPERATURE DISTRIBUTION

Since in the present mixed convection problem, buoyancy aids forced convection in dissipating the heat from both the channel walls, it would be interesting to separate out the effect of buoyancy on temperature distribution along the left and right channel walls. Fig. 6 shows the local non-dimensional left and right wall temperature profiles drawn with this objective. The study is made for a fixed input of $q_v = 2 \times 10^5 \text{ W/m}^3$, $k_s = 0.25 \text{ W/m K}$, $AR = 12$, $Ri_w^* = 250$ and $\varepsilon = 0.05$. Curve 1 pertains to the case where the effects of both free and forced convection are taken into account, while curve 2 is plotted with the result obtained by artificially suppressing the role of free convection in the present problem. As can be noticed, there is a decent influence brought in by buoyancy in a forced convection configuration with both the left and right wall local temperatures coming down due to its presence. Quantifying the above observations, in the case considered here, the local left wall temperature at the channel exit comes down by 63.58%, while the right wall temperature at the corresponding location drops down by 61.63% upon considering buoyancy in the forced convection configuration.



(a)



(b)

Fig. 6: Exclusive effect of buoyancy on non-dimensional peak channel temperature in different regimes of mixed convection

This clearly highlights the importance of including buoyancy in thermal load calculations in the entire mixed convection regime, in general, and while working in a free convection dominant environment, in particular.

9. CONCLUDING REMARKS

The effects of aspect ratio, modified Richardson number and surface emissivity on local and peak temperatures are explored. The results thus obtained elucidated the gross errors that creep in upon ignoring radiation in this kind of problems. A significant drop both in the left and right wall temperatures is noticed with the flow regime transforming from free to forced convection dominance owing to an increasing share in convection heat dissipation with its radiation counterpart remaining unaltered. The peak temperature the channel assumes decreases with increasing surface emissivity in all the regimes of mixed convection, with the degree of decrease noticed to be more apparent in the free convection dominant regime when compared to that in the forced convection dominant regime. This is attributed to an augmented convection activity in the forced convection dominant regime that overrides the role of radiation. The buoyancy has been seen to play a fairly significant role in deciding both the local and peak channel temperatures in the entire mixed convection regime.

REFERENCES

- [1] Elenbaas, W., "Heat dissipation of parallel plates by free convection", *Physica*, IX, 1942, pp. 1–28.
- [2] Quintiere, J., and Mueller, W. K., "An analysis of laminar free and forced convection between finite vertical parallel plates", *ASME Journal of Heat Transfer*, 95, 1973, pp. 53–59.
- [3] Watson, J. C., Anand, N. K., and Fletcher, L. S., "Mixed convective heat transfer between a series of vertical parallel plates with planar heat sources", *ASME Journal of Heat Transfer*, 118, 1996, pp. 984–990.

- [4] Barletta, A., and Zanchini, E., "Mixed convection with variable viscosity in an inclined channel with prescribed wall temperatures", *International Communications in Heat and Mass Transfer*, 28, 2001, pp. 1043–1052.
- [5] Gururaja Rao, C., Balaji, C., and Venkateshan, S. P., "Conjugate mixed convection with surface radiation in a vertical channel with symmetric and uniform wall heat generation", *International Journal of Transport Phenomena*, 5, 2003, pp. 75–101.
- [6] Li, R., Bousetta, M., Chénier, E., and Lauriat, G., "Effect of surface radiation on natural convective flows and onset of flow reversal in asymmetrically heated vertical channels", *International Journal of Thermal Sciences*, 65, 2013, pp. 9–27.